



Evaluation of the Existing Geometric Proportions in the Beauty of Historical Bridges of East Azerbaijan from Safavid to Pahlavi

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Received 28 July 2023; revised 22 October 2023; accepted 02 January 2024

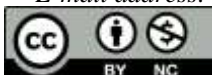
Research Article

Abstract

The role of proportions in architecture is undeniable throughout history, and the researcher has succeeded in introducing a proportional evaluation system with 11 stages and 9 proportional systems including: basic and multiple modules, proportions The golden rectangle, golden spiral, circles following the ratio of 1:618/1, Platonic rectangle, radical ratios, golden ratio, Le Corbusier's modular system and the proportions of Caen and Shako have been analyzed proportionally. This research has been carried out with the aim of extracting and classifying the existing geometric proportions in the selected bridges from the Safavid to Pahlavi periods, as well as classifying and prioritizing them. The causal-comparative research method and the strategy of answering the comparative question, which are scored based on the relevant observations and based on the observation of the extraction package and the visible items, are then entered into the SIGMAPLOT software to check the factor contribution and present the regression relationship. The linear regression relationship $Y=ax+b$ is presented. The observations are collaborative and with the approach of compiling a balance sheet, and the sampling of bridges is targeted and with entry and exit criteria. The results show that the greatest role in creating the beauty of the selected bridges of East Azerbaijan is the golden ratio with a value of (1.000) and the least is related to Le Corbusier's modular with a value of (0.195).

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Publisher: Islamic Azad University, Yazd Branch.

Keywords: Geometric Proportion; Selected Historical Bridges; Safavid to Pahlavi; Linear Regression

1. Introduction

Different views of architecture have existed as long as humans have existed and even before that, which has seen many imaginations. In this process of transformation and smooth evolution, the presence of some metamaterials has made them eternal and divine in the valley of architecture. Since the birth of architecture, geometry has been, is and will be. The knowledge of geometry, like many human sciences, has a long history, which has always been used in architecture to enhance the material and convey the spirit, meaning and special effect. (Saki and Pakzad, 2014). This type of use for creation by mankind can be influenced by the thoughts and ideas of philosophers and great thinkers, as Plato says that God is an "engineer" and before him, Pythagoras and after him Plotinus also believed that mathematics is because of its sensible area (and not Perceptible) has a theological aspect and the principles of God created the world based on an amazing mathematical order based on two right triangles (Ching, 1998).

A bridge is a phenomenon that moves the environment and landscape around it. Building a bridge is a symbolic movement; (Tahbaz, 1988). In addition to removing people's need to travel, it makes it possible to overcome an obstacle. Therefore, the history of the origin of bridges is very important; This history is rooted in the footprints of the first human who tried to cross a calm river. Iran's ancient bridges are also among engineering and artistic masterpieces, in general, in architectural works and technical works, Iranians have never been separated from each other, art and science, but these three have always been hand in hand. given and with the efforts of builders and engineers, he created a work that was rich and new in every respect. (Kozlova, 2016). By observing these rhythmic works, one can understand the way of thinking of Iranians and their relationship (Stevanović, 2013).

He understood its parts with each other and the unity of the elements and considered the material forms as an embodiment of this body of thought, bridge building is an ancient spark and in all the several thousand years that have passed since the formation of human societies on the earth until the industrial revolution of Europe in the 19th century, bridge engineering and architecture have never been separated from each other (Tsigichko, 2007). The architecture of the bridge is a combination of a structure that is affected by the river bed, the strength of the ground and the amount of water passing under it, both in normal times and in floods every few years. In every bridge, four main forces are active in interaction, which are tensile force, compressive force, bending force and shear force. In the ancient world, Iranians, Romans and Chinese were the leaders of other nations in bridge building. Iranians, the reason for living in a wide variety of climates, have been one of the nations that built durable and beautiful bridges. How effective are each type of geometric proportions in the formation of these bridges?

2. Research Background

In this research, in order to avoid procrastination in writing and speech, as well as abbreviated writing in the presentation of the conducted researches, an attempt has been made to categorize the results in the form of Table 1.

Table 1 Research history of Iranian authors

Row	Author/authors	Research title	Publication year	Conclusion
1	Reza Meshkini Asl	Investigation and identification of West Azarbaijan and East Azarbaijan bridges	2001	During the Safavid period, due to the expansion of the road network and a lot of luck towards architecture, bridge building enjoyed significant progress.
2	Amir Shah Karami, Seyyed Abdul Azim	Reading Khaju bridge engineering	2006	Paying attention to engineering aspects in the design of buildings such as Khajo Bridge has changed the perspective and analysis in researches, and modeling obtained from historical examples can show a prominent role in the architecture and development of our country.
3	Naderi Far, Hamid Reza, Ahmadi Barouq, Hamid Reza	Semantic geometry and its crystallization in the structures of Islamic art (with an emphasis on the art of mosque architecture)	2010	It examines the mystical and Quranic meanings of Islam that are manifested in the geometry of mosques.
4	Ansari, Mojtabi, Nejad Ebrahimi, Ahad	The process of intervention in Iran's historical and cultural monuments with a value-oriented policy approach	2010	By studying the geometry and proportions of the building, the decorations and shapes used, the materials and the style based on which the motifs and geometric shapes are placed together, they have come to the conclusion that the proportions examined in addition to the decorations in the building in the structure Its geometry has also been of interest.
5	Mahmoudi, Mahnoush, Chaideh, Ali	The application of mathematics in Iranian architecture (investigating the role of geometric proportions in the entrance decorations of the houses of the old texture of the historical city of Dezful)	2010	Examining the way geometry and architecture are connected in the brick decorations of inscriptions and the effect of brick dimensions in the formation of this connection in the entrance gates of traditional houses in the historical city of Dezful and providing a suitable model for the revival of brickwork and the design of new decorations.
6	Bamanian, Mohammad Reza, Okhovat, Haniyeh, Beqai, Parham.	Application of geometry and proportions in architecture	2010	By examining the concept of geometry and proportions from different perspectives, as well as different historical periods and numerous examples, she has provided solutions in terms of aesthetics.
7	Maysam Mirian	The role of fractals in geometry,	2011	Proving the similarities between Islamic motifs and fractal geometry products in the forms presented in the mentioned article, which can be

		mathematics and its relationship with Islamic motifs in Iranian buildings and mosques		considered as a suitable stimulus for new research areas.
8	Mojtaba Rezazadeh Ardabili, Mojtaba Mojtaba Sabet Fard	Recognizing the application of geometric principles in traditional architecture Case study: Palace of the Sun and its hidden geometry	2012	Realizing the geometric feature of the work and its creative application, along with other characteristics and concepts, can help to revive the authentic Iranian identity in architectural works.
9	Mona Dioj Polly	The role and position of bridge building in Safavid period architecture	2015	The role and place of bridge building in the Safavid era architecture, the importance of building bridges, due to its general utility aspect, is the prosperity and comfort that it provides for the general public; Therefore, the most important factor in bridge construction is the resistance and stability of the building against natural and human destructive factors.
10	Hassan Karimian, Saman Sidi	Geometry and geometric proportions in the construction of domes of Safavid mosques in Isfahan	2018	The results of the investigations indicate that the Safavids paid special attention to proportions in architecture and converted the square design under the dome into a circular dome design. During this period, the geometry became simpler and discrete domes became more popular.
11	Johannes Wallner & Helmut Pottman	Geometric calculations for free form architecture	2011	Geometric computing has recently found a new field of applications, that is, various geometric problems that lie at the heart of the rationalization and design processes informed by the construction of free-form architecture.
12	Maria J. Zychowska	Architecture of bridges	2015	In this article, some bridge structures from the ancient aqueducts to the newest rivers have been investigated. Their main purpose is function and efficiency rather than aesthetic considerations, and yet from a time perspective, they still impress with their beauty and perfection.
13	Prof Nadja Kurtovic Folic	Participation of architects in bridge aesthetics	2015	Introduction Examining the interface between architecture and engineering is a complex issue. The responsibilities of engineers and architects often overlap. Both professions are inseparable in the design and construction of structures such as bridges. Architects design the space to meet the needs of the client as well as the aesthetic appearance. The main responsibility of engineers is to ensure that the design is safe and complies with all appropriate structural codes.

3. Theoretical Foundations

3.1. Fit

Proportion, which in Euclid's view refers to the quantitative comparison of two similar things, has been the basis of the creation of the whole nature, including the heavens and the earth, and especially humans. Proportions have always been used in different periods and ancient civilizations in the design of buildings. Proportion in an object always makes the object look more balanced

(Hejazi, 2005). Proportions have been used in architecture from materials and materials, climatic conditions, technical and executive factors to the thoughts governing people's lives, and space has been designed as a suitable background for human growth and excellence (Akkach, 2005). Proportion is one of the basic principles of an artwork that expresses the harmonious relationship between its components (Ansari et al., 2011: 46). In another definition, proportionality is: the relative and analogical relationship between different parts and the whole of an element. Proportion, while being a determining factor for harmony, is one of the issues that has always been discussed in architecture. Proportion is a subjective value and can only be checked in relation to the shape. Proportion in architecture means a ratio that expresses the relationship between two or more sizes (Grotter, 2004: 360).

Measuring the size of two things produces a ratio. According to Euclid's theory: quantitative comparison refers to two similar things. While proportion is said to be the equality of proportions. Proportions are a set of ratios; a ratio is a comparison of two qualities or quantities such as size or amount (Critchlow, 1989).

Therefore, ratios are considered to represent a unit of a difference or difference. In the field of architecture, proportions include the comparative ratios of various quantities and qualities of heterogeneity, and hence its understanding is more complicated. If we take the fit as an example of the activity of perception based on the recognition of difference (Carrier, 2005: 9).

3.2. Geometry

The word geometry refers to the science of the properties and relationships of quantities such as points, lines, surfaces or volumes in space and the way parts of a particular object fit together (Concise Oxford English Dictionary, 1999). The Arabic word for geometry is the word "size" in Persian. Geometry means size and shape. It is one of the principles of mathematical sciences and it is a science in which the states of quantities and sizes are discussed (Dahkhoda Dictionary). Geometry is a word that Islamic scholar chose against the Greek word "geometry", which consists of two parts "geo" meaning earth and "meter" meaning measurement (Dore and Murphy, 2013). The first confirmed record of geometrical knowledge and its relationship with astronomy, man and music dates back to ancient Greece, especially, to the time of Pythagoras and Plato. Perhaps the Greeks were the first to popularize geometry, although being the first does not mean being the first to discover it (Elam, 2001). Ibn Sina considers geometry to be the science of knowing the position of lines, shapes, surfaces and proportions. In other words, different definitions of geometry all emphasize the relationship between geometry and shapes and proportions (Fletcher, 2019). Therefore, the science of geometry is a powerful tool that has enabled the architect to measure spatial proportions and create balance, order and beauty. Geometry plays an essential role in the design of architectural buildings (Imamoglu, 2000). From the point of view of external performance, the use of geometry as art to create shapes, patterns and proportions recalls the great architect of the universe (God). Therefore, the art of geometry is the key to creating a connection between the building and the ideas that the builder has in his mind. (Josephine, 2017).

Table 2 Definitions of Geometry and Proportion (Hejazi, 2005: 17-44).

Given definitions of geometry and proportion		
Size and shape		Concept
Studying in space and imaginable shapes and objects in this space	Geometry	Mathematics
The proper relationship between the	Proportion	

components with each other and with the whole work		
The proper relationship between the components with each other and with the whole work	Visual Arts	
Creating visual beauty, an inseparable part of the architectural space	Architecture	

The study of geometry in ancient civilizations indicates that geometry is divided into two practical and theoretical categories:

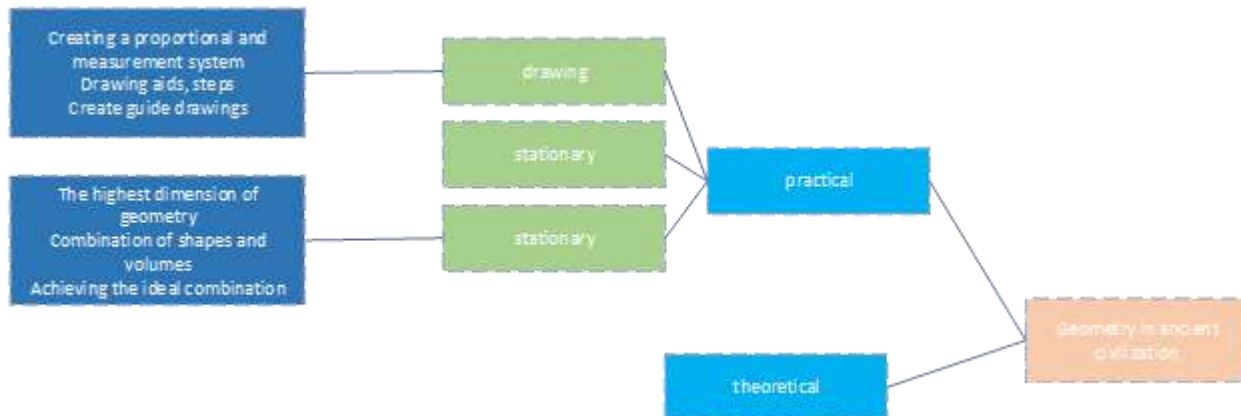


Fig 1 Geometry in ancient civilizations (Ansari et al., 2011: 53).

Geometry is both quantitative and qualitative in nature. Its chemical dimension regulates the order and structure of design forms. Its quality essence determines the dimensions of the design forms and presents the representation of the world order as a visual representation (Zhu, 2012: 1741). Each pattern or geometric shape, when considered from the perspective of its symbolic meaning, displays an echo of unity and a reflection of values and principles within the larger frames beyond unity (universal unity). Seyed Hossein Nasr states that geometry and rhythm represent a single doctrine that is central to Islam, according to which Islamic art is based on mathematical dimensions and shows the dimensions that are the heart of Islam (Elam, 2001).

Geometry is the main design of the creation of all shapes. It is a science that is related to numbers in space with 4 main levels: the first is the science of arithmetic (pure numbers), like any dimension and size, it is a geometric measurement (Imamoglu, 2000: 11). The second level is figures in space, which displays dimensional geometry. They reflect meanings and "ideas". The third level of time figures is the basis of music. The fourth level is the figures in time and space that represent the universal cosmology (Dabbour, 2012).

Dimensional geometry and primitive roots that consider the most beautiful dimensions of beauty are included. Thus, beauty, for Muslim artists, is an objective and self-descriptive truth, the essential nature of beauty, as Plato said, "beauty is the glory of truth" (Akkach, 2005).

3.3. Theories Related to Proportional Systems and Measurement Units

All theories of proportions aim to create a sense of order between the components of a visual composition. The proportional adjustment system creates a set of visual fixed ratios between the components of a building and also between the components and the whole. Proportion regulation

systems go beyond functional and technical determinants of architectural form and space, and present aesthetic arguments about themselves (Kozlova, 2016)

The theories related to systems of proportions and units of measurement can be examined in two ways:

- 1- Geometry and proportions in Iranian-Islamic thought
- 2- Geometry and proportions in global thought

3.4. Geometry and Proportions in Iranian-Islamic Thought

Iranian Golden Proportions is the name given by Pirnia engineer to the ratio of the sides of a rectangle enclosed in a regular hexagon, and among the published articles and books, these ratios are considered to be involved in the formation of Iranian proportions (Abolghasemi, 2005). The things mentioned by him are the formation of three doors based on the ratio of the sides of a halved rectangle inside a regular hexagon and also the formation of courtyards based on a complete rectangle surrounded by hexagons. Utilizing the knowledge of geometry and proportion in Iran's architecture has had a special place both before Islam and in the Islamic era (Haji Ghasemi, et al., 2012). With the advent of Islam, since mere imitation of nature did not have a prominent base in Islamic culture, a single and unified view emerged in the combination of abstract art and separated from matter and nature, which saw the world as transcendent and did not follow nature completely (Bamanian, Okhovat, and Baqaei, 2010: 171). The system of Islamic proportions is based on the geometric properties of square, double square, equilateral triangle and pentagon, which are equal to Asam numbers and the world of balanced proportions or Iranian proportions is $1/41=\sqrt{2}$ and $1.73=\sqrt{3}$ and $\sqrt{5}$ and $(1.118=\sqrt{1.25}$ and $\sqrt{5.2})$ (Stevanović, 2013) which are derived from $\sqrt{2}$ and $\sqrt{3}$ are created (Ayat Elahi, 1998).

The use of $\sqrt{2}$ and $\sqrt{3}$ ratios in the ancient architecture of Iran, as well as the use of pimon in the architecture of Iran after Islam, indicate the use of the precise system of adjusting proportions in Iranian architecture (Bamanian, Okhovat, and Baqaei, 2010: 175)

$\sqrt{2}$ represents the geometric shape of a square and the resulting shapes are surface representations, while $\sqrt{3}$ represents the geometric shape of a triangle and the resulting shapes are volume representations. If we consider a square with a side of one unit and make an arc equal to its diameter with a ruler, the larger side of the obtained rectangle is equal to the diameter of the square, i.e. $\sqrt{2}$. With the obtained rectangle diameter, you can draw a $\sqrt{3}$ rectangle, and with a $3\sqrt{}$ rectangle diameter, you can draw a $\sqrt{4}$ rectangle, and this process can continue. Such rectangles are called dynamic rectangles. The $\sqrt{3}$ rectangle is called a Platonic rectangle that forms an equilateral triangle (Nikghadam, 2012).

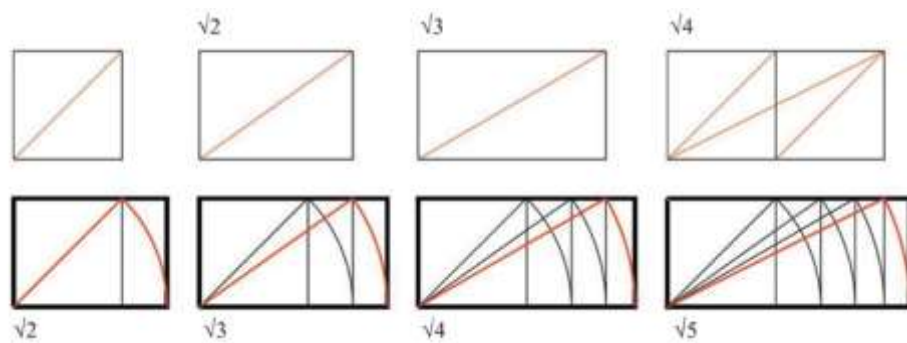


Fig 2 Consecutive structures in proportional rectangles based on square diameter (Source: Stevanović, 2013).

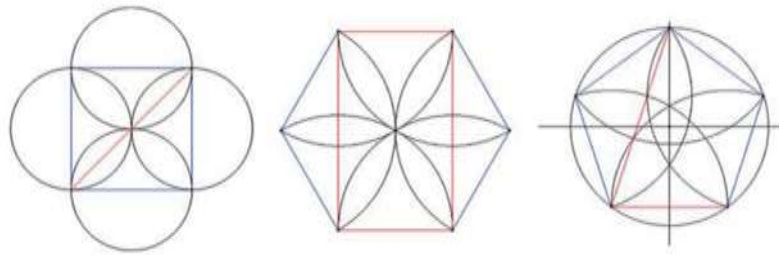


Fig 3 Roots of dimensions $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ (Source: Stevanović, 2013).

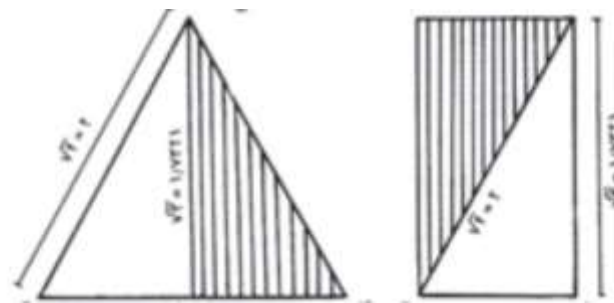


Fig 4 Platonic rectangle (Source: Stevanović, 2013).

3.5. Golden Ratio

The golden ratio (also called the golden ratio, golden mean, divine ratio, divine proportion, sacred ratio, or simply the Φ ratio) is a transcendental ratio found in fundamental forms such as plants, flowers, viruses, DNA, shells, Planets and galaxies are found. $\sqrt{5}$ is a ratio that paves the way for the principle of ratios called the golden ratio (Stevanović, 2013). The golden ratio is a constant ratio that is derived from a geometrical relation and like the number π and other similar constants in numerical components, it is the "nominal" number. The numerical value of the golden ratio, which is called Φ , is $\Phi = 1/6180339000$ or $(\sqrt{5}+1) = \Phi$ (Lawler, 1989: 95). $\sqrt{5}$ represents the pentagonal geometric shape and the resulting shape is the expression of nature and it is obtained from the combination of 5th regular shapes of decagonal, icosahedron and the like, which is widely used in Islamic architecture and examples of these proportions and their combinations in Nature is found, among other things, in the proportions of the human body (Tsigichko, 2007).

The golden ratio has unique features:

$$\dots (((((\dots / 1+1) / 1+1) / 1+1) / 1+1) / 1+1) = \Phi$$

The ratio of the parts in the pentagon and the five vertices (five-pointed star) which were sacred to Plato and Pythagoras. Also, the twelve faces of the Platonic volumes are the twelve faces in the shape of a pentagon, which includes the golden ratio (Hejazi, 2005). For this reason, Plato considered this shape to be equivalent to the universe. In all five vertices, each larger (or smaller) part is related to the ratio T , so that a series of powers of the golden ratio are automatically generated with consecutive ascending (or descending) powers: Φ^4 , Φ^5 , $^3\Phi$, Φ , Φ^2 ,

The general rule of Esan's body is also made up of the golden ratio (Haji Ghasemi et al., 2012). Examples of the applications of these proportions in Iranian architecture before Islam are the long middle porch of the Kasri Palace in Tisphon, which followed the $\sqrt{3}$ ratio, and the Apadana Palace in Persepolis, which followed the $\sqrt{2}$ ratio. He named in the post-Islam era, the ratio of $\sqrt{2}$ was used in the Imam Mosque of Isfahan. Sarvestan Palace has golden proportions (Dabbour, 2012). The ratio of 1.118, which is obtained from the golden ratio, was used in pre-Islamic architectural proportions. This proportion can be seen in the length and width of Sarostan Palace and Kasri Palace, which are Sassanid palaces (Fletcher, 2019).

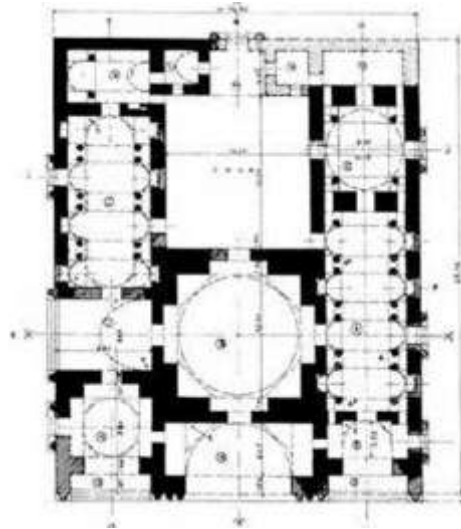


Fig 5 Sarostan Palace, application of proportions 1.118 (length to width ratio) (Source: Fletcher, 2019).

3.6. Geometry and Proportions in Global Thought

Geometry and proportions in global thought can be divided into 4 categories:

- a- Golden proportions
- b- Renaissance theories
- c- Le Corbusier modular
- d- Human proportions (Tahbaz, 1998).

a. Golden Proportions

The ancient Egyptians used proportions that they called theological proportions. Later, these proportions were called the divine proportion by Vitruvius, an Italian architect of the second century. But at the end of the 19th century and the beginning of the 20th century, when gold became the standard of economic measurement, these proportions became popular with the term golden proportions (Dabbour, 2012). The law of golden divisions of line segment by Euclid, a prominent Greek philosopher and mathematician, in the third century BC. discovered. Also, after some time, the Greeks realized the dominant role that the golden ratio played in the proportions of the human body (Stevanović, 2013). Therefore, they reflected these proportions in the building of their temples. In this ratio, a line is divided into two unequal parts, where the ratio of the length of the smaller part to the larger part is equal to the ratio of the length of the larger part to the whole line. The golden ratio is the ratio of 1 to 61803.1. (Zychowska, 2015). Whenever a shape or volume has allegorical or acceptable dimensions and sizes, it is called proportional or having golden sizes.

Sublime and golden ratios have become common in every culture according to the beliefs and likes of that culture and thought, and it has proven its beauty due to the multitude of uses (Amir Shah Karami, 2006). Proportions in their general form rely on the science of geometry and mathematics in their place and in their specialized form, they have an undeniable value in the basics of understanding art and are considered as fundamental considerations (Carrier, 2005: 9). $\sqrt{3}$, $\sqrt{2}$ and ... and Fibonacci numbers (Carrier, 2005: 71). Architects also used this law during the Renaissance period. Le Corbusier set his modular system based on golden proportions. The golden ratio that has been widely used in Islamic architecture is the ratio obtained from pentagonal dimensions (Ansari, Okhovat, and Taghvaei, 2011: 71).

If three points are on a straight line, the ratio of the large segment to the small segment is equal to the ratio of the length of the entire line segment to the length of the large segment (Vitruvius called this geometric ratio theological ratio in the second century AD). Lahuti ratio: It is a ratio that divides a line segment into two proportional parts so that the ratio of the smaller part to the larger part is equal to the ratio of the larger part to the whole line segment (Abolghasemi, 2005).

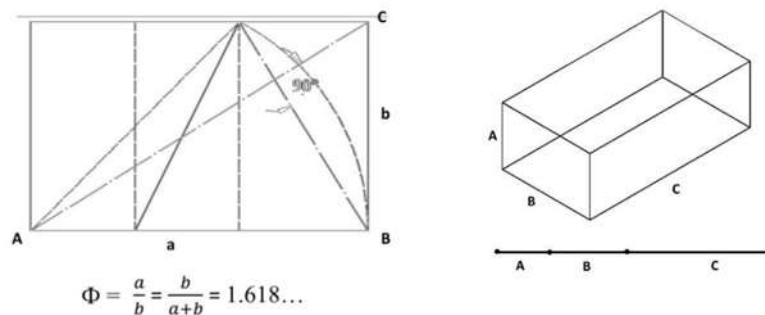


Fig 6 Golden proportions (Source: Ching, 1998: 300)

The golden ratio of the line segment is represented by the 21st letter of the Greek alphabet, T. Phi Dias, a Greek sculptor, studied the golden ratio in detail, and for this reason, this ratio is also known as Phi (Φ) (Kashifpour, 2009). Dividing the line segment into two proportional parts can be used to make: 1) golden rectangle and spiral, 2) golden pentagon, 3) golden triangle.

Golden rectangle and spiral:

In making the golden rectangle, like $\sqrt{2}$, the index square is used. With the difference that to draw a golden rectangle, Kozlova, N. (2016) we make an arc equal to the diameter of the square from the diameter of half the index square, the obtained point shows the place of formation of the golden rectangle, where the points (c, e, f) are the same as points A, B, C there are golden lines (Kozlova, 2016).

Golden pentagon:

As mentioned earlier, the number $\sqrt{5}$ represents the pentagonal geometric shape. Now, the regular pentagon enclosed in the circle is the golden pentagon, which forms the golden regular decagon with another pentagon, upside down (Ayat Elahi, 1998). In a pentagon, the diagonals are divided into two proportional parts like the golden section (Bemanian et al., 2011).

Golden Triangle:

As mentioned, the $\sqrt{3}$ rectangle is called the Platonic rectangle, which forms an equilateral triangle and, in this definition, the isosceles triangle that exists in the golden pentagon drawing is called the golden triangle, which can be divided into two other golden triangles (Wallner and Pottmann, 2011).

Fibonacci sequence:

The mathematics of the golden ratio was found by Leonardo Fibonacci, the founder of the "Fibonacci sequence", a series of numbers found many times in the natural world. This sequence follows the rule that the next number is the sum of the previous two numbers, as follows: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, etc. (Elam, 2001). If we divide any number in the Fibonacci sequence by the number before it, say 144.5 or 89.5, the answer is always close to 1.61803, which is the golden ratio value. The Fibonacci sequence, or characteristic description of the golden ratio, can be seen anywhere in the forms of nature, music, and art. One of the mathematical products is the sacred spiral ratio, which is commonly found in nature (Rawles, 1997).

b. Renaissance Theories

Pythagoras realized that the sound harmony of the Greek music system can be expressed by the following simple expansion 1, 2, 3, and 4 and their ratios as 1:2, 1:3, 2:3, and 3:4. This ratio led the Greeks to believe that they had found the key to the mysterious harmony that pervaded the universe (Hejazi, 2005). Pythagorean believed that everything is arranged according to numbers. Later, Plato completed the science of calculating Pythagorean numbers as the science of proportion. He squared and cubed this simple numerical expansion to obtain double and triple expansion (Hejazi, 2005). According to Plato, these numbers and their ratios not only understood the harmony of sounds in Greek music, but also showed the harmonious structure of the world. Renaissance architects, believing that their buildings should belong to a higher order, referred to the Greek system of mathematical proportions. The Greeks believed that music is geometry translated into sound, Renaissance architects believed that architecture is mathematics translated into spatial units (Mahmoudi and Chaideh, 2010). By applying the Pythagorean theory about the intermediate ratios of intervals in the steps of Greek music, they completed the infinite progression of ratios that formed the basis for the infinite ratios of their architecture. These sets of ratios not only showed themselves in the dimensions of a room or a facade, but also appeared in the interconnected proportions of a string of space or the whole plan (Mirian, 2011). As can be seen in the figure below, using the golden ratio and $\sqrt{2}$, this set of ratios can be seen in the dimensions of a room, facade, and in the interconnected proportions of the space or the whole plan, which Palladio 7 types of the most proportional He suggested the rooms in 4 books about architecture. It should be noted that rectangle is the most common shape in design, which is expressed by the ratio of width to length, such as: 2:3, 3:5, 8:5 and so on (Rezazadeh and Mojtaba Sabet Fard, 2012).



Fig 7 Palladio's theory about seven types of the most appropriate rooms (Source: Mays, 2008)

c. Le Corbusier's modular system

Le Corbusier considered the measurement tools of Greece and Egypt, which were part of the mathematics of the human body and were the source of harmony governing human life, very rich, for this reason, his measurement tool, the modular system, is based on mathematics (golden ratio and Fibonacci series) and completed the proportions of the human body (functional dimensions of

the building) (Meshkini Asl, 2001). Le Corbusier began his studies in 1942, and in 1948 he published a book called *Modular, a Human-Scale Pion of General Application in Architecture and Mechanics* (Elam, 2001). The second volume, *Modular 2*, was published in 1954. Le Corbusier looked at the modular not only as a set of numbers with a fixed agreement, but as a measurement system that was subject to lengths, sides, and volumes, and could establish human proportions and scale everywhere (Tsigichko, 2007). The main grid consisted of three sizes: 43, 70, and 113. (Their ratio was adjusted according to the golden ratio) (Ansari and Nejad Ebrahimi, 2010: 50). Le Corbusier calculated the length of an average human, which was equal to 183 hundredths of a meter, and obtained his proportions. These ratios are on the one hand: 86, 140, 226 (with raised hand) and on the other hand, 70, 130, 183 (up to the top of the head) (Kozlova, 2016).

$$113=70+43$$

$$183=70+113$$

$$226(113*2) = 43+113+70$$

d. Human proportions

The system of adjusting proportions according to human proportions is based on the dimensions and proportions of the human body. In this system, they use the theory that the form and spaces in architecture include and occupy the human body and therefore should be determined by its dimensions (Rezazadeh and Mojtaba Sabet Fard, 2012). If the size of the middle part of the body to the sole of the foot is considered as one unit, the height is equal to 1.618, which is the same number as Φ (Kashifpour, 2009: 88). According to the Holy Qur'an, man has within himself all that is reflected in the world "the best proportion" (Mirian, 2011). Man is the core of God's creations; He has the most harmonious proportions, reflecting the harmony of the divine. "Indeed, we created man with the best form" (proportion). Leonardo da Vinci illustrated the geometric dimensions of the human body by showing that humans clearly display the dimensions of the golden ratio in their bodies based on the ratio of 1.618 (Mahmoudi and Chaideh, 2010). The Vitruvian Man painted by Da Vinci based on Vitruvius, who wrote that human dimensions should be related to architecture. Vitruvius believed that if human dimensions could join with buildings, they would be complete in their geometry (Guenon, 1995).

According to Robert Lawler; "The human body includes its dimensions in all important geometric geodesic sizes and functions...the dimensions of the ideal human lie at the center of a circle of constant cosmic relations" (Lawler, 1982).

From the comparison of Le Corbusier's modular proportional system and human proportions, it can be seen that from dividing Le Corbusier's numbers to each other using the golden ratio of the line segment ($BC/AB = AC/BC$), that is, $2.52 = 113.70 = 43.70$, which is approximately It is equivalent to $\sqrt{5} = 2.23$ and in the system of human proportions, human height is considered as the golden number Φ , which is equal to $(\sqrt{5}+1)/2$ or $\Phi = 1.6180339000$ (Hejazi, 2005).

From the summation of the material presented about proportional systems, it can be concluded that the proportional adjustment systems can be examined on two scales in Iran and the world, and it can be concluded that the Iranian-Islamic proportions can be called balanced proportions $2\sqrt{5}$ And $3\sqrt{5}$ and $5\sqrt{5}$ introduced that among these $5\sqrt{5}$ is introduced under the title of golden ratio and proportions on a global scale include two divisions of golden proportions and human proportions, (Elam, 2001) which renaissance theories are based on golden proportions and the modular system Le Corbusier deals with both the golden ratio and the human ratio, and in general with the golden number Φ , which is equal to $(\sqrt{5}+1/2)$, and it can finally be claimed that all proportional systems

with one of the $4\sqrt{2}$ criteria and $\sqrt{3}$ and $\sqrt{5}$ and the golden number Φ can be analyzed (Meshkini Asl, 2001). Therefore, this result can be seen in Figure 8.

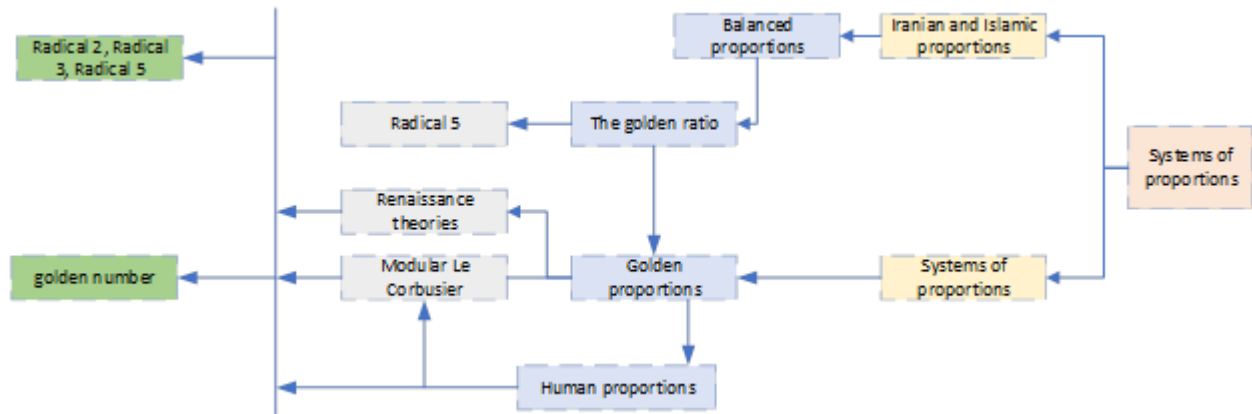


Fig 8 Diagram of proportional systems

4. Research Methodology

The research method in this research is based on the comparative approach in investigating the geometric proportions. The nature of the research is developmental-applied that the sampling of the bridges is based on the purposeful sampling and in the selection part of the participatory observers, the selection approach is snowball with the entry criteria (Gall, Borg, and Gall, 2018; Cresswell and Clark, 2019). The statistical population for the selection of participatory observers is expert people. Space users are used to score the degree of beauty, and the PC modeling system is used to strengthen the applied geometry and determine the application of proportions with each other. Sampling of random users is calculated by the amount of 384 numbers and based on the upper limit of Morgan's table. The observation is a predetermined package with a balance sheet and includes three parts: decorations, shapes, and overall form. It is divided, then based on the contribution of each geometry in its formation, a number of 1 to 5 is given. In the next step, the effect of each from the geometries and beauty of these bridges, a questionnaire with a Likert scale is designed and provided to space users. The results are entered into the ORIGINPRO software and analyzed with inferential statistics. Validity with the CVR formula for the observation tool and questionnaire is done for 20 experts, which is 0.76 and 0.74, respectively. The reliability of the questionnaire was estimated with Cronbach's alpha, which is 0.83.

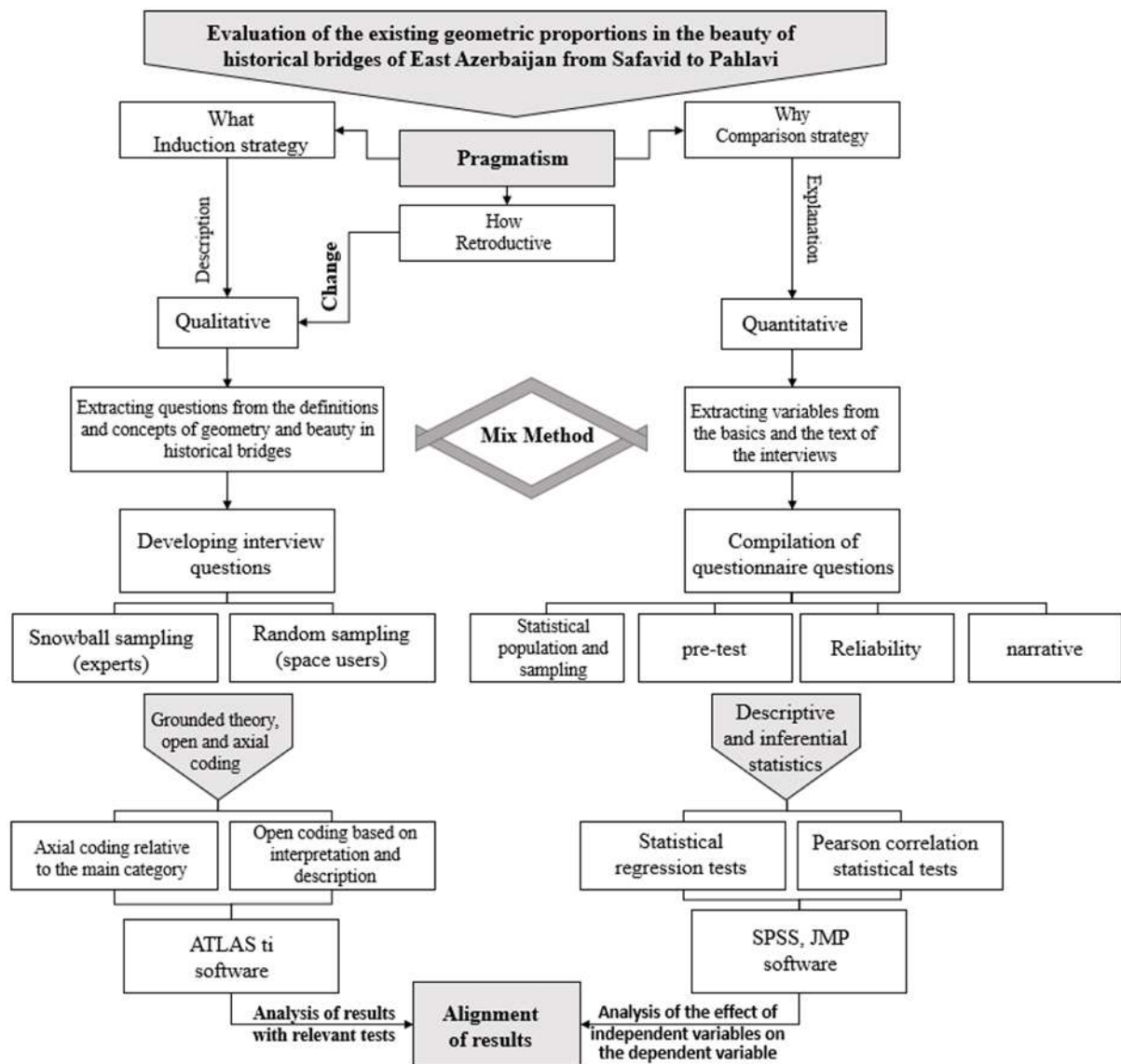










Fig 9 Research process




4.1. Measurement Range

This research uses the bridges of East Azerbaijan as the unit of analysis to select the samples. To achieve suitability for analysis, the preference system with Kendall's W is used, and items below 0.5 are removed. In the table below, the results of introduced bridges and brief explanations are given.

Table 3 Introduction of measurement range and selected bridges

Topic	Construction Period	Description	Kendall Coefficient	Image
Aji chai Bridge	Safavid	With 16 openings, it is 105 meters long and five meters wide, and it has been reconstructed and restored in every period of damage caused by natural disasters such as river flooding or human destruction.	0.746	 A photograph of the Aji Chai Bridge, a long stone bridge with multiple arches, situated in a dry, dusty environment with mountains in the background.
Tabriz stone bridge	Qajar	Tabriz stone bridge is another historical bridge of this metropolis, which dates back to the Qajar period. This bridge is located on Chaiknar Street and opposite the Stone Bridge Mosque on the Mehraneh River. It is said that a person named "Haj Azim" is the founder of the construction of the stone bridge.	0.824	 A photograph of the Tabriz Stone Bridge at night, illuminated by warm yellow lights, showing its stone structure and arches.
Khoda Afarin bridge	Safavid	Khoda Afarin bridge was built in Khoda Afarin city and on Aras river. This bridge consists of two separate bridges, one of which is broken and only half of it remains, but the other bridge has not been damaged. This bridge is located near Khoda Afarin Dam in "Sari Jalu" village. The distance between these two bridges is 800 meters and they are located in two regions of Azerbaijan, namely "Qara Dag" and "Karabagh", and they are different from each other in terms of engineering and architecture. One of the bridges, which has 11 spans, is made of white stone, its height is 12 meters and its length is 130 meters.	0.723	 A photograph of the Khoda Afarin Bridge, a long stone bridge with multiple arches, situated in a river valley with green hills in the background.

Qari Bridge	Safavid	It is one of the bridges that are still in use today and is reminiscent of the old generation of bridges and the not-so-distant world. Another old bridge in Tabriz is the "Qari" bridge, which was previously known as "Pacheragh."	0.596	
Sahib al-Amr Bridge	Safavid	Next to the mausoleum and mosque of Sahib Al-Amr, built during the time of Shah Tahmasab Safavid in 1405 AH, the soldiers of Sultan Murad IV destroyed it, which was restored by Mirza Mohammad Ibrahim, the Minister of Azerbaijan. In the earthquake of 1193 AH, the Sahib Al-Amr complex was destroyed. It was rebuilt in 1208 AH by Jafar Khan Dinbeli.	0.611	
Ancient bridge of five springs	Safavid	Other important bridges of East Azerbaijan are the ones that established the connection between the worlds of human societies in the ancient era and were placed in the shape of a crescent on the Safi river and are currently used by passers-by.	0.810	
Middle Girl Bridge	Safavid	22 kilometers of the Middle Zanzan road related to the 8th century AH is one of the most important crossings of the international silk road (east to west) and Dokhtar Malekan bridge related to the Safavid period are considered as other important and ancient bridges in East Azerbaijan.	0.481	
Paul Mardegh	Safavid	Mordagh Bridge has three spans, the middle span is bigger than the spans on the two sides, and its gable arch is made with native red bricks. The bridge itself is built from local stones of this region, which were extracted from the "Sanjan" mine. In this bridge,	0.377	

		there are also two small arch springs for the purpose of passing flood waters and for more beauty. In the past, Mordagh Bridge was the connection between Maragheh and Hashtroud cities.		
Shahr chai Bridge	Safavid	Shahr Chai Bridge is located 12 kilometers south of the city of Mianeh, near Taze Kend village and on the river Shahr Chai. Detailed information about the construction date of this bridge is not available, but according to the architectural style and materials used in it, it can be attributed to the Safavid era, because it is very similar to the 33 bridges of Isfahan, but it has a simpler shape.	0.830	
Qizlar bridge	Safavid	The Qizlar bridge was built on the Mardagh river, which is located between the village of "Qala Chouk" and "Qoli Kandi" in Malkan. This bridge is very important because of the type of arches and stone and brick facade, and it is full of travelers and tourists during the holidays. It is said that the construction date of the bridge dates back to 951 AH and Safavid rule, and it has been renovated several times. This bridge is still very important in the passage of the people of the surrounding villages.	0.603	
Bilankoh Bridge	Safavid	The Bilankoh bridge is located in the Bilankoh neighborhood of Tabriz, located on Abersan street and Sheikh Kamal alley. The bridge was built on the "Esberez" river, which originates from the Sahand mountains and finally flows into the Mehraneh river. This bridge, which is made of brick and stone, belongs to the Qajar period, but there are traces of another bridge near it, which	0.522	

		archaeologists attribute to the Safavid period.		
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The removed bridges include Bilankoh, Murdagh Bridge, Dokhtar Miane Bridge, which after being removed due to the low score of the expert board, they inquire about the reason. They point out that the main reason for this is a combination of these things (improper repair), (inaccurate information), (lack of easy access), lack of familiarity with the expert panel.

4.2. Qualitative Findings

At this stage, after observing the researchers and evaluating them in the efficiency of proportions, percentages are started between each proportion used in different parts.

Table 4 Percentage evaluation of the available geometries of selected bridges based on focused focal observations

The name of the selected bridge	Rand coefficients of the base modulus		Golden rectangle		Golden spiral		Circles 1 to 618/1		Platonic rectangle		Radical ratio				golden ratio		Modular Le Corbusier		Ken and Shako	
	Outline	Components	Outline	Components	Outline	Components	Outline	Components	Outline	Components	Outline	Components			Outline	Components	Outline	Components	Outline	Components
												Radical 2	Radical 3	Radical 5						
Qari Bridge	15%	37%	21%	8%	16%	35%	17%	5%	19%	21%	24%	12%	14%	10%	41%	29%	21%	16%	0	12%
Sahib al-Amr Bridge	18%	24%	15%	12%	21%	19%	11%	14%	14%	11%	21%	14%	9%	8%	17%	21%	0	6%	10%	17%
Ancient bridge of five springs of Bonab	14%	18%	21%	14%	21%	18%	16%	12%	4%	9%	14%	8%	19%	18%	24%	10	0	7%	3%	4%
Tea City Bridge	8%	19%	18%	14%	15%	14%	18%	2%	1%	18%	12%	7%	19%	18%	26%	12%	1%	9%	6%	6%
Bridge girl	14%	11%	14%	9%	5	17%	15%	24%	20%	19%	15%	16%	18%	16%	18%	26%	5%	6%	12%	14%

Based on the findings of this stage, it is clear that the golden ratio with a value of 29% in the components and 41% in the overall design has the largest contribution in the formation of the geometric proportions of Qari Bridge. In Sahib Al-Amr bridge, in the components, the Rand factor of the base modulus is 24% in the general plan and 21% in the components, the highest is related to the radical ratio. In the ancient bridge of five springs of Bonab, the golden ratio with 24% in the general plan and the radical ratio of 3 for the most important components have played a role in the formation. In Shahr Chai bridge, the golden ratio is the highest with 26% in the general design and in the components related to radical 3 with a value of 19%. In Dokhtar bridge, the circles are 1 to 1.618 with a value of 24% in the components and in the general design, the most related to the rectangle. Platonic with a value of 20%.

Paul Aji Chai is removed from the selected samples due to departure from the centrality of the data as well as skewness of the results.

4.3. Inferential Statistics

At this stage, in order to find out how effective each of the mentioned geometries is in the beauty of the selected historical bridges, a visual questionnaire is distributed. In this part, recursion should be used to perform factorial component. For this purpose and to choose the type of regression, the

correlation matrix is used. The results show that they do not have a linear component with each other, and it is necessary to use multivariate regression.

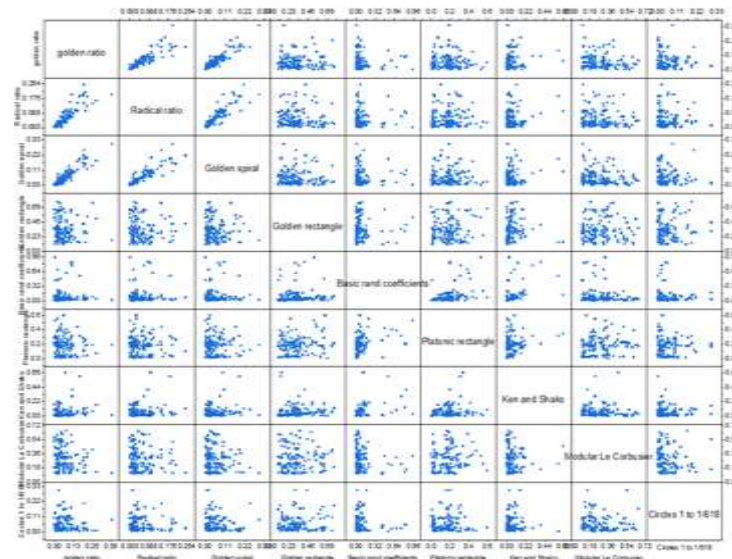


Fig 10 Correlation matrix of geometric proportions in selected bridges

Based on this, multivariate regression is used, and SIGMAPLOT is used for ease of doing this. The results are presented in the table below.

Table 5 Multivariate regression, step by step types of geometric proportions in creating the beauty of bridges

Meaningful	t	β	F	Coefficient of determination	Variable
0.008	571/44	0.762	217/314	0.867	Platonic rectangle
0.007	365/31	0.372	147/523	0.195	Modular Corbusier
0.006	255/31	0.872	381/852	0.580	Circles 1 to 1/618
0.001	479/58	0.685	921/298	1.000	golden ratio
0.003	982/21	0.597	257/247	0.612	Golden spiral
0.001	134/11	0.436	321/644	0.656	Golden rectangle
0.009	425/24	0.852	523/845	0.645	Basic rand coefficients
0.009	132/23	0.665	254/754	0.316	Ken and Shako
0.018	121/48	0.213	541/124	0.715	Radical ratio

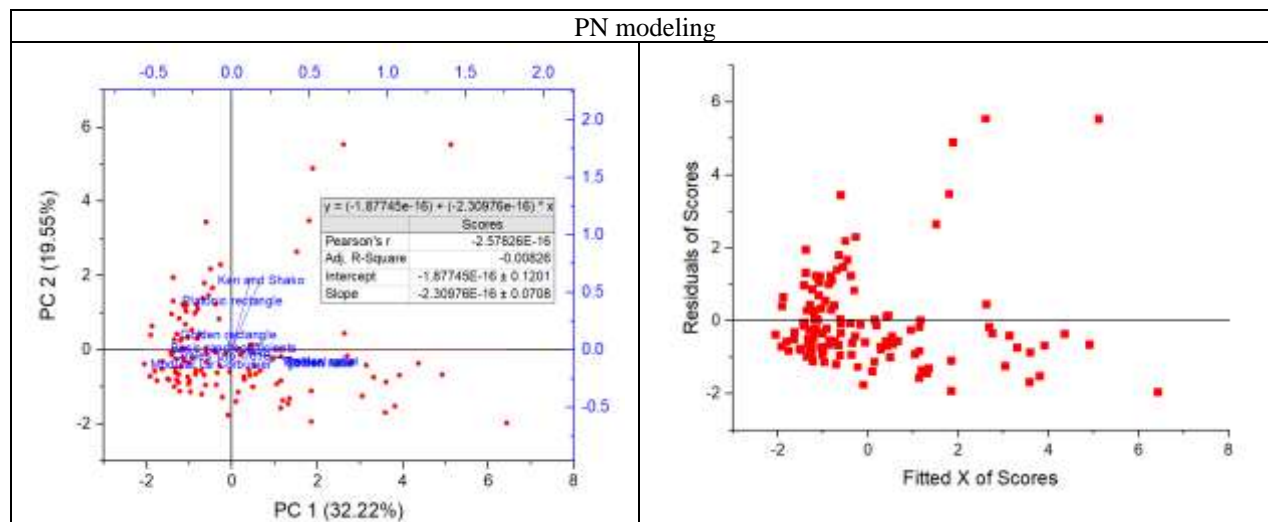
Based on the findings of the regression model, it is clear that the greatest role in creating the beauty of the selected bridges of East Azerbaijan is the golden ratio with a value of (1.000) and the least is related to Le Corbusier's modular with a value of (0.195). Therefore, the regression model of applying geometric proportions to achieve beauty is as follows;

$$Y = (0.867) \text{ Platonic rectangle} + (0.195) \text{ Modular Locorbusier} + (0.580) \text{ Circles 1 to 1/618} + (1.000) \text{ golden ratio} + (0.612) \text{ Golden spiral} + (0.656) \text{ Golden rectangle} + (0.645) \text{ Basic rand coefficients} + (0.316) \text{ Ken and Shako} + (0.715) \text{ Radical ratio}$$

Based on the PN modeling, it is clear that the application of the golden proportions, the radical ratios and the Platonic rectangle reinforce each other in creating beauty, and these results are the same for the base rand coefficients of circles 1.618 to 1 and the golden spiral, but Le Corbusier's

modular And Ken and Shako act separately and have different results in creating beauty.

Table 6 PN modeling of selected types of geometric proportions in the beauty of selected bridges



5. Discussion

Based on the data distribution findings and on the basis of focused observation, it is determined that the most types of geometric proportions used in bridges are related to the golden proportions, and it seems that in all periods for its overall design in one direction or in more The general and three-dimensional forms of bridges have been used most of all in the components of the most general proportions that have been used in shapes or decorations and have been used in openings, the radical proportions are known as Asam numbers. It seems that one of the reasons for the fortifications at that time is the simultaneous application of these two proportions together. Based on the inferential statistics of the factor contribution, it is determined that the radical ratio and the golden proportions are also effective in creating the beauty of bridges, or perhaps they have had the best effect, but the rest of the proportions that have historical or territorial roots are equally effective in creating beauty. They have normal limits.

The proportions of Ken and Shako and the modular proportions of Lucor Boutzia have a smaller contribution to the creation of beauty, perhaps due to the climatic distance or the time dimension. In the PN modeling, it is clear that the use of a number of proportions lead to an increase in the effect of creating beauty in these bridges. Golden proportions, radical ratios, and platonic rectangles can be considered to increase the effect in creating beauty.

6. Conclusion

In Iranian architecture, geometry has always been used to create fortifications or create beauty in decorations or overall form. Historical bridges are one of the few urban elements that, in addition to their functional form, are also responsible for transporting people. And they are also a place for people to spend time. In the past, historical bridges have been able to remain until now due to high fortifications and play a role as a beautiful urban element.

Iran's bridge-building periods can be called pre-Safavi and post-Safavi periods. The main function of bridges is crossing, but in addition to this main function, Iranian bridges are combined with special services. Another feature is creating a different space, in addition to the passage space.

Due to the structural structure of Iranian bridges and the use of pointed arches, the way of transferring forces is such that it enables the creation of empty space in the body of the bridge. The remains of the oldest bridge in Iran date back to the 8th century BC on the Aras River, which was built by the Urartians. But the most important areas where bridge building has been necessary and popular are the mountainous regions of western and northern Iran. In this area, there are many permanent and seasonal rivers. These rivers were among the major natural obstacles on the roads. In these areas, a continuous process of building bridges, restoration for road construction, and the construction of bridges in these areas was necessary. In the Safavid period, bridge building was given too much attention and famous bridges were built in this period. The Safavids were the dynasties that made deep and fundamental changes in Iran and performed outstanding actions in terms of culture and art. He was one of the most important artistic indicators in the field of growth and prosperity of Iranian architecture. This research shows that there is the use of different geometries in a bridge, but most of them used the golden ratio in the general designs and also used the radical ratio for different components. This research showed that the application of various geometric proportions together leads to the creation of beauty and harmony.

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