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International Journal of Applied Arts Studies

IJAPAS 5(2) (2020) 29–40

Visual Design Principles of Ancient Art

Mahsa Behnood[a]*

[a]*Assistant Professor, Department of Art, Faculty of Engineering, Golestan University, Gorgan, Iran*

Received 09 May 2020; revised 24 May 2020; accepted 28 June 2020

Abstract

This work presents an important investigation of design basics and the golden proportional model in Persian painted pottery motifs (late fifth-early fourth millennium B.C.) and presents a comparison among Persian, Egyptian and Grecian fundamental design basics. Generally, Persian, Egyptian and Grecian pottery motif designs have some similarities and dissimilarities in their design basics. It can be concluded that the employed elements are nearly the same for all the investigated civilizations. The techniques used to achieve the principles in Persian studied designs have been understood to be either design of Egyptian painted motif like or Grecian motif design like. The used golden proportional models were found to have distinctive variations for the different civilizations. Persian has a higher degree of diversity in golden proportional models compared resulting in more well performed/pleasant designs.

Keywords: Design Basics; Principles; Golden Proportion Model; Persia

1. Introduction

Since the early times, mankind gazed at nature and around world; countless artistic artworks have been created. In their mind, they had many designs for creating and every designer was inspired by nature. Designer created his works based on his beliefs and the place where he lived. Designer attempted continually for representing nature shapes in abstract and geometric styles. Animal motifs in the ancient civilizations of Persia, Greece and Egypt played important role in life of ancient people, religion, ritual and sacrificial rites for the goddess. Indeed, animal and human motifs had been objects to reach the fertility, fruitfulness, survival and warding off from evil. Ancient artworks were a creative solution for the mentioned desires. These designs seek to visual

* Corresponding author. Tel: +989117309910.

E-mail address: m.behnood@gu.ac.ir

communication to the viewer to give a message as content of a design. Designer employed elements arranged by principles for establishing a visual communication. Recognizing the techniques employed by Persian ancient artists (based on modern element and principles definitions) to achieve principles of design is very important in concept of design history, because our knowledge about this subject has not yet been completed, especially for the Porto-Elamite and Elamite eras (ca. 5000-2600 B.C.) where the earliest painted potteries can be found.

In addition, one of the characteristics of observed Proto-Elamite/ Elamite motifs is existence of the pleasing relationships between the design components. It seems that effective design principles and proportional models incorporated with the golden number as, a simple practical mathematic idea, were employed. Also there are long historical relationships between Art and Mathematic ideas and geometry (Le-Corbusier, 2008; Gottschall, 1989; Muller-Brockmann, 2003). The importance of mathematics ideas in art is stated by Max Bill (1949); “I am of the opinion that it is possible to develop an art largely on the basis of mathematical thinking” (Kimberly, 2011). Golden proportion as a universal law (Zeising, 1854) plays a fundamental function in the natural structure blocks (Browm, 2003) and creates pleasing relationships.

Ancient peoples thought that the golden proportion was created by God and it exists everywhere in the nature (Benjafield, 2010; Browm, 2003). Since ancient times, the principle of the proportion has been employed (Green, 1995) such as in the Sumerian arts, in Egyptian great Pyramid (Burton, 1985; Gardner, 1957; Westren Turnbull, 1956), Greek vases, pottery of Chinese and Cretan (in French) and Mycenaean (in Greek) products of the late Bronze Age (3100-2200 B.C.). Greek religious buildings have been designed based on an approximation of φ with the accuracy around 0.5% (Green, 1995). Due to the simplicity of the structure and application, the golden proportion has a considerable impact on the art and architecture of the middle ages and the Renaissance (ca. 1350-1600) (Elkins and Williams, 2008). Phidias is aware of the aesthetics of the golden proportion in constructing the Parthenon and occurrence of the golden ratio in the Parthenon is frequently reported (Browne, 1989; Bergamini, 1963; Hill, 1990; Hude, 1974; Manuel and Santiago, 1988) (Pappas, 1989; Mitchell, 1977).

The majority of the investigations to recognize the golden dividing have been conducted on the Greece, Egypt civilizations and European art in medieval period where the earliest studied evidences belong to Bronze Age about 3000 B.C. and the employing of this idea is proofed. In spite of that, has not been paid an attempt on recognizing of design basics and the golden-based model on Persian painted pottery motifs derived from Neolithic and Early Bronze ages (ca. 8000-2000 B.C.).

Existence of pleasing relationship, high degree of aesthetics, and a visual coherence in Elamite era shows there could be a cognitive preference for pleasing ratios as inspired by nature. It can be hypothesized that the designer used near or equal values to Golden ratio for element organization in their design and was aware of its existence and features. The objectives of this study are; Understanding how ancient Persian designers achieved design basics; Making clear the position of the art of ancient Persia in the history of golden ratio; and Recognizing the similarities among Persian, Egyptian and Grecian fundamental design basics.

1.1. Terminology of Golden Ratio

The golden proportion is called the “fundamental building blocks” in the natural world (Browm, 2003) but, it is not exactly identified who, for the first time has proposed the golden section concept. However, golden proportion is attributed to Pythagoras (ca. 570-495 B.C.) and Plato (ca. 428–427 B.C.), a student of Socrates, (Naini, James, and Daljit, 2006). The golden ratio According to Euclid (ca. 325–265 B.C.) is the “division into mean and extreme ratio” (Heath, 1956; Berlyne, 1971; Herz-Fischler, 1998). Euclid in his well-known book entitled with “Elements” explains the calculation method of golden ratio (Devlin, 2005). Leonardo of Pisa, known as Fibonacci, (ca. 1170-1250) presents a series of numerical that is called the “Fibonacci sequence”. This sequence number begins with 0 and 1 then each successive number obtains by sum of prior consecutive two numbers like 0, 1, 1, 2, 3, 5, 8, 13, ... There is a strong relationship between this successive ratio of this sequence and the golden proportion (≈1.618) (Dunlap, 1997). Johannes Kepler (1571-1630) demonstrates that the limit of the Fibonacci sequence is approximately equal to the golden ratio (Tattersall, 2005). Luca Pacioli, (1509) in the 16th century, uses the expression of “de divina proportione” in his book that focuses on the golden section, illustrated by Leonardo da Vinci (Crosby, 1998; Pacioli di Borgo, 2004; Kemp, 2004). Maestlin (1597) first purposes the famous golden ratio decimal calculation (Herz-Fischler, 1998). The golden ratio is often represented by other researchers as the golden section[†] (Coxeter, 1953), the golden number (Fischler, 1981), golden mean (Linn, 1974), divine proportion (Huntley, 1970), the Fibonacci number (Dunlap, 1997) and mean ratios (Smith, 1953). The sign of Phi (φ) has been taken from the first alphabet of the name of Phidias (ca. 490 BC–430 BC) who is one of the most famous ancient Greek sculptors and architects (Fowler, 1982).

2. Materials and Methods

The methodology is subdivided into 3 parts including analysis of design basics (Formal analysis), the proportional analysis and comparison. The same design basics and proportional analysis were done on the some earliest observed painted pottery motifs of Egypt and Greece civilizations to be compared with Persian motif designs. Fifty-nine ancient painted potteries (ca. 5000-2600B.C.) are examined and after ignoring repetitive and indistinct figures, a total of 38 motifs designs were selected. For comparison part, a total of 66 painted potteries were examined for the Egypt and Greece civilizations. The sources to find motifs were documents, interviews of experts in the ancient art of Persia and golden proportion, observations from libraries, research centres and museums visit (in person and virtually). Documentations and archival evidences such as books, articles and excavation reports used to provide background of the research and to highlight the gap of knowledge on this subject.

2.1. Studied Eras and Area

For ancient Persia, painted pottery motifs belong to the late fifth-early fourth millennium B.C. to 2600 B.C. or Proto-Elamite classified into three sub-periods of Period Susa I. (ca. 5000-3500), Period Susa II (ca. 3500-3100 B.C) and period Susa III (ca. 3100-2600 B.C.) (Figure 1a). For comparison section, the observed earliest Egyptian and Greece painted potteries cover the periods of ca.3650-3300B.C and ca.2500-1300B.C., respectively. The Egypt studied period is contemporary with the Pre-dynastic era ancient Egypt timeline (ca. 5300-2950B.C.). For Greece civilization, the study period is contemporary with the Late Bronze Age, ca.1600-1050B.C. (Mycenaean culture) and a small part of the Early Bronze age (ca. 3200-2000B.C.) of the ancient Greece timeline (Figure 1a).

[†] . in Latin names *sectio divina*

2.2. Term of Susa

This research deals with design motifs observed from Neolithic and Early Bronze ages excavated from Susa as ancient Persia civilization and the capital of Elamite (5000–650 B.C.) as well as the economic and cultural capital of Achaemenid empires (559–330 B.C.), located at west south Iran, around 1.17 km northwest of Ahwaz modern city. Today Susa city is located at the same place after 7000 years.

2.3. Formal and Proportional Analysis

Identification and explanation of elements and principles were done based on the definitions presented by (Hashimoto and Clayton, 2009; D'alleva, 2006; Feldman, 1972). The Kimberly Elam's geometric approaches (Kimberly, 2011) are used for constructing golden models based on the ratio of 1.618. To find used techniques, the remarks and definitions presented by (Pentak and Lauor, 2015) were used.

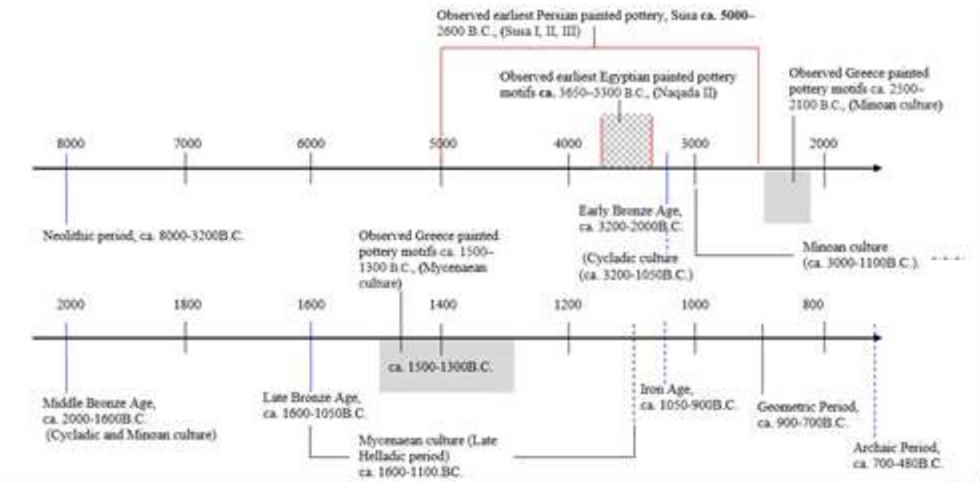
3. Results and Discussion

The animal and human motifs are the main subjects in motif designs. A variety of animals on the painted potteries in different civilizations is observed (Figure 1b). The most repeated animal motifs are Ibex, bird and horse in Persia, Egypt and Greece, respectively. The presence of animal and human motifs on ancient painted potteries is expected, because in all the ancient civilizations, animals play a key role in daily living of people. In Egyptian and Grecian motif designs there are not a given symbol associated with the motif compared to Persian motif.

Results of elements analysis show that most lines were straight, bold and rigid in diagonal and vertical directions. A difference in the used lines thickness is seen (Figure 1b). It is interesting that the ancient Persian and Egyptian designers used the diagonal and spirals dominant orientations to illustrate the feeling of flowing and present the motion (Figure 1: P7, 13, 25, 26, 28; E30, 32, 33, Figure 2: b2, b3). There are a number of Egyptian painted potteries including just spirals motifs. The vertical lines were drawn to show a feeling of more stability in design while diagonal lines were used to present a sense of dynamism and moving feeling. In Persian motifs designs the dominant vertical direction is frequently observed (Figure 1: P1-4, 14, 18, 22, 23), while in Greece and Egypt diagonal and spiral direction are dominant. Persian motifs mostly have clear boundaries between elements and if all the contours are traced, there will not be any confusion. In addition, curving forms used to emphasize on natural shapes such as horn and body of animal and to give a sense of softness and flowing (Figure 1: P1-4, 7, 10, 26, 28, E:24, 29, 32, G36-38). Combination between curvilinear and rectilinear forms in which one of them was dominant created the focal area (Figure 1: P1-4, 11, 15, E31, 33, G39). All motifs created monochrome or silhouette associated with using efficient negative space. Negative spaces in some studied motifs have had an equally importance in comparison with the subject matter. For example, designer used positive and negative space (square) to create a pattern to design the symbol of farm land (Figure 1P: 1, 4, 11). In all designs the object and background (yellow, brown, black and red color) can obviously be distinguished.

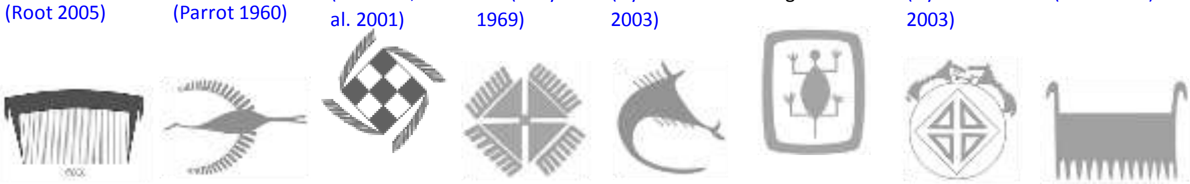
Principles of balance, repetition, contrast, harmony, unity, emphasis and dominance were found on the studied designs. The dominant balance had been symmetry and near-symmetry in Egypt and Persia civilization with horizontal and vertical axes (e.g. Figure 1: P1-6, 9, 10, 14-17, 19, 21-23, 27). The radial balance is observed on Persian painted pottery motifs, results in closeness and unity.

(a)

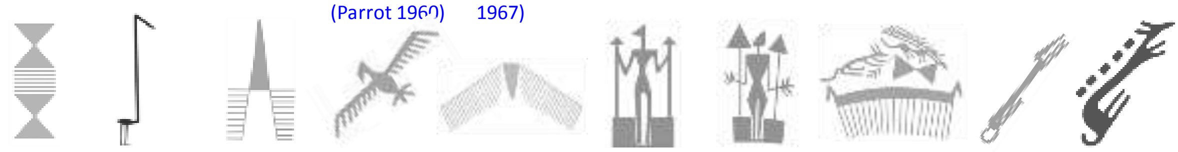


(b)

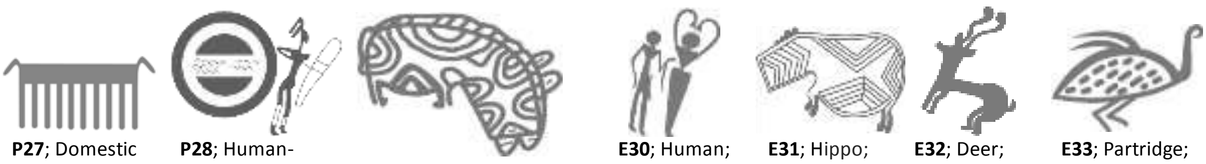
P1; Ibex; 4000BC. (Root 2005)	**P2**; Ibex; 5000BC. (Parrot 1960)	**P3**; Ibex; 4000BC. (Clibborn, et al. 2001)	**P4**; Ibex; 5000-4000 BC. (Dury 1969)	**P5**; Ibex; 4500BC. (Ayatollahi 2003)	**P6**; Sheep with sign of irrigation	**P7**; Stork; 4500BC. (Ayatollahi 2003)	**P8**; Snake; 5000-4000 BC. (Root 2005)



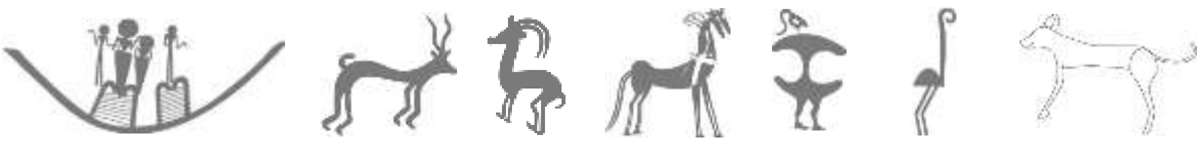
P9; Sheep with symbol of water; 4000BC. (Root 2005)	**P10**; Stork; 4500BC.	**P11**; bird-comb, sing of earth; 4000BC. (Parrot 1960)	**P12**; Chalipa; 4500BC. (Pope 1967)	**P13**; scorpion ;5000-4000 BC. (Root 2005)	**P14**; tortoise; 4000 BC. (Root 2005)	**P15**; Dog and Horse; 4500 BC. (Hole and Wyllie 2007)	**P16**; Ibex; 5000BC. (Parrot 1960)



P17; Stylized Ibex; 3000BC.	**P18**; Crane; 5000BC. (Parrot 1960)	**P19**; Vulture; 4000BC. (Root 2005)	**P20**; Vulture; 4500BC. (Pope 1967)	**P21**; geometric Vulture; 4000BC. (Root 2005)	**P22**; Human; 5000-4000BC. (Root 2005)	**P23**; Human; 4000BC. (Parrot 1960)	**P24**; Ibex, Sheep, Wolf 4500BC. (Pope 1967)	**P25**; Dog; 5000BC. (Parrot 1960)	**P26;** Dog; 4500BC. (Ayatollahi 2003)



P27; Domestic Sheep; 4500BC (Pope 1967)	**P28**; Human-irrigation sign 4000BC.	**E29**; Hippo; ca.3650–3500 B.C.	**E30**; Human; ca. 3650-3300BC.	**E31**; Hippo; ca.3650–3500 B.C.	**E32**; Deer; ca. 3650-3300BC.	**E33**; Partridge; ca. 3450–3350 B.C



E34; Human; ca. 3450–3300 BC. Metropolitan Museum

E35; Deer; ca. 3650-3300BC.

E36; Goat; ca. 3500–3300 BC.

E37; Horses of Chariot, ca. 1400–1370 B.C

G38; Human; ca. 1500-1300 B.C.

G39; Flamingo, ca. 3650–3300 BC.

G40; Dog; ca. 2500-2100 BC. Athens National Archaeological Museum

Figure 1 Comparative chronological chart, the earliest painted pottery motifs in different civilizations observed, gray zone refer to Greece (ca. 2500-1300B.C.), (Based on (Mcintosh, 2009; Neer, 2011; Pedley 2007, Perlès 2001, Béatrix 2000), Hatch area refers to Observed earliest Egyptian painted pottery motifs ca. 3650–3300 B.C., (Naqada II) (based on (Mcintosh, 2009; Roger and Klaas, 2004; Dorman, Russmann and Lilyquist, 1983; Lioyd, 2004; Béatrix, 2000), (b) selected studied motifs, Abbreviation: [First letter of Civilization][motif number]; like **P**=Persian.

Symmetrical balance as the simplest type of balance represents the feeling of permanence, stability and strength. Ancient Persian designers position the visual importance area (like symbols of earth and fertility) or focal points on the central axis of design where symmetrical balance can be found (e.g. Figure1: P1-4, 6, 9); while this characteristic is not observed in Egyptian and Greece motif designs.

The repetition of element was one of the main principles in all investigated civilizations. Repetition of line with variation in size is observed in all studied civilizations to form a rhythmic feeling (Figure 1: P10, 11, 19, 20), movement (e.g. Figure 1: P11, E32-33) and illusion of depth (Figure 1: P1-5, 24-26, 28, E29-31, 34-37, 40, G40).

The repetition of lines and forms is applied to achieve the principle of overall unity (e.g. Figure 1: P29, 31, E33). Ancient Persian also employed the closeness and continuation techniques (e.g. Figure 1: P1-4, 8, 12, 15, 24) to present the visual unity into while, closeness is rarely observed in Grecian and Egyptian painted potteries. Also, visual spiral direction on the whole design created unity and a visual linkage amongst the elements. For Persia, Egypt and Greece, superimposed golden spirals on the motif designs have good coincidence, implies the occurrence of continuation (e.g. Figure 2: b1, b2, b3; Figure 3: d1, d2, d3).

Persian designers used the methods of exaggeration (e.g. Figure 1: P1-5, 11), the positive and the negative spaces placement (e.g. Figure 1: P1, 4, 11), curved and angled shapes arrangement (e.g. Figure 1: P1-5, 11, 15) and viewpoint changing (e.g. Figure 1: P7, 10, 18, 22-26, 28) to achieve the emphasis. Through changing in form and size break the overall design pattern (e.g. Figure 1: P6, 9) to achieve the emphasis principle. Ancient Egyptian and Persian designers, utilized methods of pattern breaking, size variation and viewpoint change to present a visual emphasis while in studied Grecian motifs changing the viewpoint was not seen.

In Persia, depth illusion or space is formed using the techniques of repetition with difference in size, multiple view point and oblique projection (Figure 1: P26, 28). Multiple viewpoints include side-plan (Figure 1: P7, 10) and side-front views (Figure 1: P18, 22, 23). Egyptian artists used similar techniques except oblique projection (Figure 1: E31-34). Grecian artists applied the repetition with variation in size to present a feeling of depth (Figure 1: G38-40).

4. Proportional Analysis

Overall, proportional analysis indicates that in Persia and Egypt a variety in golden proportional models is observed both in the vertical and horizontal directions. Proportional models of [1:φ], [1: φ: φ^2] (Figure 2: a2, a6-a8, a10, a12) and [1:φ: φ^2: φ^3: φ^4] (Figure 2: a5) were found in horizontal direction on Persian motifs. The golden vertical proportional models of [1:φ], [1: φ: φ^2] (Figure 2: a2, a3, a11), and [1: φ: φ^2: φ^3: φ^4] (Figure 2: a7) are seen in Persian designs. It is interesting that the focal points (as symbols like earth, fertility, water) were localized based on the closed golden proportion models (Figure 2: a1-a4, Figure 3: a1).

In the same way, in Egyptian motifs the golden models of [1:φ], [1:φ:φ^2] (Figure 3: b1, b2, b5, b6), [1: φ: φ^2: φ^3] (Figure 3: b3, b7) and [1:φ], [1:φ:φ^2] (Figure 3: b1-b2, b5-b7) are recognized in vertical and horizontal directions, respectively. In Greece the horizontal golden proportional models of [1:φ], [1: φ: φ^2] (Figure 3: c1, c2) and [1:φ: φ^2: φ^3] (Figure 3: c3) and vertical models of [1: φ: φ^2] (Figure 3: c1, c2), [1: φ: φ^2: φ^3] (Figure 3: c3, c4) and [1: φ: φ^2: φ^3: φ^4] (Figure 3: c2) were observed.

It can be concluded that the ancient designers applied the golden proportional models to achieve satisfying relations amongst elements. A well agreement exists between golden proportional models and position of the focal areas on design for Persia.

Generally speaking, results can increase body of our knowledge regarding to the basics of design and existence of golden proportion on motifs in ancient Persia, the late fifth-early fourth millennium B.C. to 2600 B.C. This research highlights the role of art of ancient Persia in history of golden proportion and presents some clues for existence and application of the golden proportion in ancient Persia.

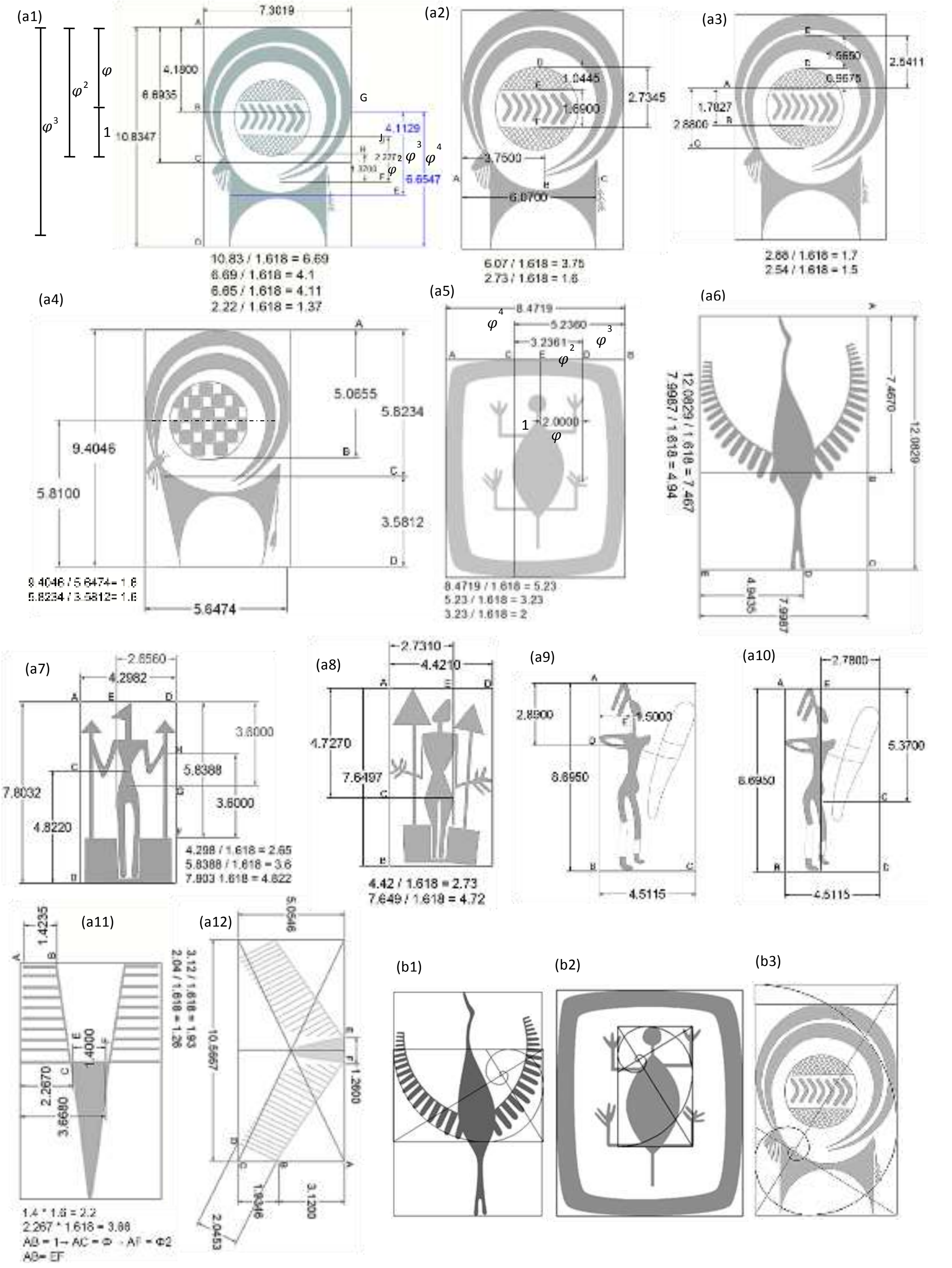


Figure 2 Selected motifs; (a1-a12) existence of golden-based proportional models in the studied painted potteries derived from ancient Persia, (b1-b3) selected results to show relatively good agreement between golden spirals and element of motif designs.

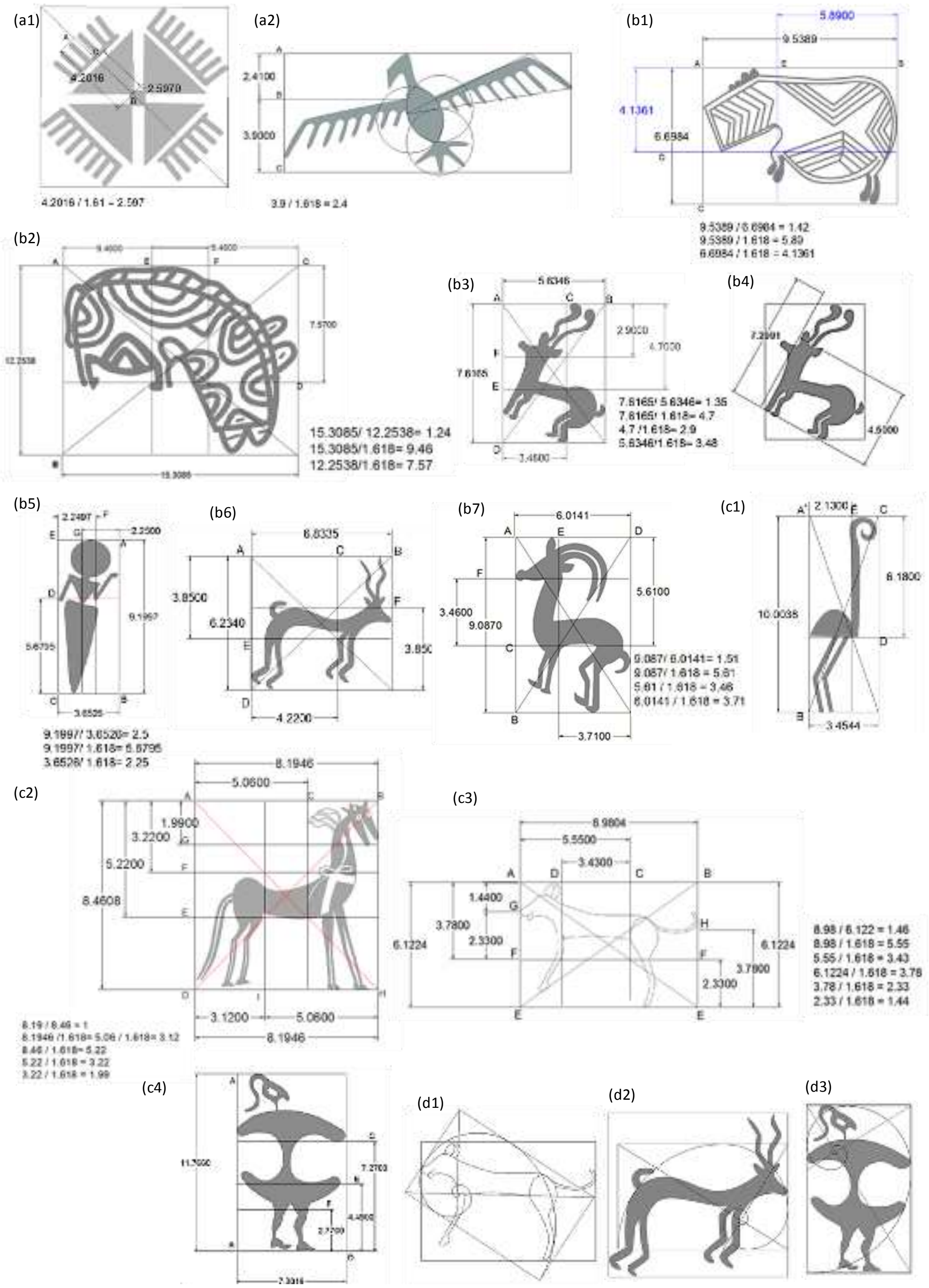


Figure 3 Selected results; (a1-a2) existence of golden-based proportional models in the painted potteries derived from ancient Persia, (b1-b7) golden proportional models in Egyptian motif designs, (c1-c4) golden proportion in some Grecian motif designs, and (d1-d3) selected results to show relatively good agreement between golden spirals and whole work of designs.

References

Ayatollahi, H. (2003). *The history of Iranian art* (Haghshenas, P. E. S. Trans.). Center for international-culture studies.

Béatrix, M. R. (2000). *The Prehistory of Egypt from the First Egyptians to the First Pharaohs* (1 edition ed.). Wiley-Blackwell.

Benjafield, J. G. (2010). The golden section and american psychology, 1892-1938. *Journal of the History of Behavioral Sciences, 46*(1), 52-71.

Bergamini, D. A. (1963). *Mathematics*. new York: Time Incorporated.

Berlyne, D. E. (1971). *Aesthetics and psychobiology*. New York: Appleton-Century-Crofts.

Browm, D. (2003). *The Da Vinci Code*. New York: Doubleday.

Browne, M. W. (1989, September 5). "Impossible" form of matter takes spotlight in study of solids. *New York Times*, p. CI & CII.

Burton, D. M. (1985). *The history of Mathematics: An Introduction*. Boston: Allyn and Bacon.

Clibborn, E., Madina, A., Sutcliffe, M., & Shahbazi, A. S. (2001). *The splendor of Iran. Ancient times* (Booth Clibborn ed., Vol. 1). (N. Pourjavady, Ed., P. E. A. French-English: M. Barr, & H. Mashayekh, Trans.). London: Booth Clibborn.

Cook, T. A. (1979). *The Curves of Life*. Dover Publications; Trade Paperback Republication edition.

Coxeter, H. S. (1953). The Golden Section, Phyllotaxis, and Wythoff's Game. *Scripta Mathematica Journal*, *19*, 135-143.

Crosby, A. W. (1998). *The measure of reality: Quantification and Western society, 1250–1600.* New York: Cambridge University Press.

D'alleva, A. (2006). *look! The Fundamentals Of Art History*. London: Prentice Hall; 2nd edition.

De Morgan, J. (1990). *Fouilles à Suse en 1897-1898 et 1898-1899, Excavations at Susa in 1897-1898 and 1898-1899*. Mission archéologique en Iran, Mémoires I, (Archaeological Mission in Iran, Memories I).

Devlin, K. J. (2005). *The Math Instinct: Why You're A Mathematical Genius (Along With Lobsters, Birds, Cats, And Dogs)*. New York: Thunder's Mouth Press.

Dorman, P., Russmann, E. R., & Lilyquist, C. (1983). Egyptian Art. *The Metropolitan Museum of Art Bulletin*, *41*(3), 3-65.

Dudley, U. (1999). *Die Macht der Zahl: was die numerologie uns weismachen will (The Power of numbers, what does the numerology us believe)*. Springer.

Dunlap, R. A. (1997). *The Golden Ratio and Fibonacci Numbers*. World Scientific Publishing.

Dury, C. J. (1969). *Art of the ancient near and Middle East* (first edition in Dutch ed.). (Brown, A. Trans.). Holle Verlag GMBH. German.

Elkins, J., & Williams, R. (2008). *Renaissance Theory* (first ed.). Routledge.

Feldman, E. B. (1972). *Varieties of visual experience* (2nd edition ed.). Prentice Hall.

Fischler, R. (1981). How to find the “golden number” without really trying. *Fibonacci Quarterly, 19*, 406-410.

Fowler, H. D. (1982). A generalization of the golden section. *Fibonacci Quarterly, 20*, 146-158.

Gardner, M. (1957). *Fads and Fallacies in the Name of Science*. New York: Dover.

Gasche, H. (1973). *Ville Royal de Suse: vol I: La poterie elamite du deuxieme millenaire a.C, Mission archéologique en Iran, Mémoires 47, (The pottery of the second millennium BC Elamite, Archaeological Mission in Iran, Memories), (in French)*. Memories.

Ghirshman, R. (1968). Suse au tournant du III au II millenaire avant notre ere. *Arts Asiatiques, 17*, 3-44.

Gottschall, E. (1989). *Typographic Communications Today*. The MIT Press.

Green, C. D. (1995). All that glitters: a review of psychological research on the aesthetics of the golden section. *Perception, 24*(8), 937 – 968.

Hashimoto, A., & Clayton, M. (2009). *Visual Design Fundamentals: A Digital Approach*. Charles River Media; 3rd edition.

Heath, T. L. (1956). *The thirteen books of Euclid’s elements, Vol. 2*. New York: Dover.

Herz-Fischler, R. (1998). *A mathematical history of division in extreme and mean ratio*. New York: Dover. (Original work published 1987.).
Hill, F. S. (1990). *Computer Graphics*. New York: Macmilan.
Hole, F., & Wyllie, C. (2007). The oldest depictions of canines and a possible early breed of dog in Iran. *Paléorient, 33.1*, 175-185.
Hude, C. E. (1974). *Herodoti Historiae*. Londini: Oxonii.
Huntley, H. (1970). *The Divine Proportion* (1st.Edition ed.). Dover Publications.
Kemp, M. (2004). *Leonardo* (First Edition ed.). Oxford: Oxford University Press, USA.
Kimberly, E. (2011). *Geometry of design* (2 ed.). New York: princeton Architectural press.
Le-Corbusier. (2008). *Towards a New Architecture* (first edition: 1931 ed.). BN Publishing.
Linn, C. (1974). *The Golden Mean*. New York: Doubleday, Garden City.
Lioyd, A. B. (2004). The Late Period (664–332 B.C.). In I. Shaw, *In The Oxford History of Ancient Egypt* (pp. 369–94). New York: Oxford University.
Manuel, G., & Santiago, A. (1988). An unexpected appearance of the golden ratio. *The College Mathematics Journal, 19*, 168-170.
Margaret, C. R. (2005). *This Fertile Land Signs + Symbols in the early arts of Iran and Iraq*. Michigan: Kelsey museum publication.
Mcintosh, J. (2009). *Handbook to Life in Prehistoric Europe*. USA: Oxford University Press.
Mitchell, J. E. (1977). *The Random House Encyclopedia*. New York: Random House.
Muller-Brockmann, J. (2003). *The Graphic Artist and His Design Problems* (3rd edition ed.). Ram Publications.
Naini, F. B., James P., M., & Daljit S., G. (2006). The enigma of facial beauty: Esthetics, proportions, deformity, and controversy. *American Journal of Orthodontics and Dentofacial Orthopedics*, 277-282.
Neer, R. T. (2011). *Greek Art and Archaeology: A New History, c. 2500-c. 150 B.C.* (1st edition ed.). Thames & Hudson.
Ogden, R. M. (1937). Naive geometry in the psychology of art. *American Journal of Psychology, 49*, 198-216.
Pacioli di Borgo, L. (2004). *De divina proportione*. Italy: Silvana.
Pappas, T. (1989). *The Joy of Mathematics*. San Carlos: Wide World Publishing.
Parrot, A. (1960). *Sumer*. (Emmons, S. G. Trans.). Thames and Hudson.
Pedley, J. G. (2007). *Greek Art and Archaeology* (4th ed.). Prentice Hall.
Pentak, S., & Lauor, D. (2015). *Design Basics* (9th ed.). Cengage Learning.
Perlès, C. (2001). *The Early Neolithic in Greece: The First Farming Communities in Europe (Cambridge World Archaeology)* (1st ed.). Cambridge University Press.
Pope, A. (1967). *A survey of Persian art* (Vol. VII). Asia Institute of Pahlavi University.
Roger, S. B., & Klaas, A. W. (2004). *Chronological Systems of Byzantine Egypt* (2nd ed.). Brill Academic Publication.
Root, M. C. (2005). *This Fertile Land Signs + Symbols in the early arts of Iran and Iraq*. Kelsey museum publication.
Schoot, A. V. (2001). Kepler’s search for form and proportion. *Renaissance Studies, 15*(1), 59-78.
Smith, D. E. (1953). *History of Mathematics, Vol. 2 (Special Topics of Elementary Mathematics)*. (Vol. II). New York: Dover.
Tattersall, J. J. (2005). *Elementary number theory in nine chapters* (second ed.). Cambridge University Press.
Westren Turnbull, H. (1956). The great mathematicians. *Newman's The World of Mathematics*, 75-168.
Zeising, A. (1854). *Neue Lehre van den Proportionen des meschlischen Körpers*. Leipzig: preface.
Zeising, A. (1884). *Der goldne Schnitt* (The golden section). Halle: Blockmann.